International Journal of Innovation, Management and Technology, Vol. 1, No. 3, August 2010
ISSN: 2010-0248

Relevancy of Fuzzy Concept in Mathematics

Priyanka Kaushal, Neeraj Mohan and Parvinder S. Sandhu

***Abstract*— The basic objective of mathematics education is to stimulate one's intuition and logical thought process. Since the intuition is fuzzy, one cannot be confined to two–valued logical thinking. There should be some natural exposure to develop a one's mathematical thinking in fuzzy -valued logical way so that one may be able to express their intuition in terms of multi valued logic instead of two valued logic to make the mathematics system more flexible and adaptable.**

***Index Terms*—Fuzzy Logic, Fuzzy set, Classical Mathematics, Multi-valued Logic, Two-valued Logic.**

I. Introduction

The Language which is widely used in Mathematics is two valued logical language. Whereas, in our daily life including our mind and Language, we can not restrict ourselves to two-valued logic, there is always a need for multi valued logic.

According to Magnus [1], Nietsche was the first to point out the fuzzy concept. But classical mathematical logic divided the world into "yes and no", "white and black", "true and false". Actually, since not all sentences involve concepts that are subject to two valued logic, we have to deal with many kinds of sentences that come from our lives and thoughts, and ordinary mathematical logic cannot handle them thoroughly.

There are many areas in which a natural language is preferred rather than the formal language of mathematics e.g. the fields like psychology, pedagogy, sociology, epistemology, cognitive science, semiotics and economics.

Logic plays a very important role in mathematical system. In fact, multi valued logic can help to construct a more justified mathematics instead of two valued logic. This will help to express our mind and life more precisely and completely. If the theory of existing mathematics can be stated in terms of multi valued logic then our students can be benefitted. Thus our aim of mathematical education towards the development of one's intuition would be fulfilled.

In this paper, first we consider the limitation of, so called, two-valued mathematics and two-valued mathematics education, and then we introduce the need of fuzzy concept which is an infinite-valued logic incorporated using fuzzy sets, which also generalizes the two-valued logic. Next, we discuss the origin of fuzzy concept.

Priyanka Kaushal is working as Senior Lecturer in Department of Mathematics of Rayat and Bahra College of Engg. & Bio-Technology for Women, Mohali, Punjab (INDIA).

Neeraj Mohan and Parvinder S. Sandhu are associated with Department of CSE & IT, Rayat and Bahra Institute of Engg. & Bio-Technology, Mohali, Punjab, India. Email: erneerajmohan@gmail.com, parvinder.sandhu@gmail.com

We give some background educational philosophy on the problem of why mathematics education should be improved. As one prescription to this problem, we are going to suggest the adoption of fuzzy logic in the mathematics curriculum. Finally we summarized some useful applications of the fuzzy concept.

II. Limitation of Classical Mathematics

A mathematical system is composed of an undefined concept, an ordinary set, algebra of sets, relation, operation, rule of reasoning, logical axiom, non-logical axiom, definition and theorem[4].

But mathematics is more than a collection of theorems, definitions, problems and techniques: it is a way of thought. Wittgenstein [2] says that what we call mathematics is a family of activities with a family of purposes. According to him mathematics as consisting of motley of human activities driven by a range of human goals, intentions and purposes. In addition, there is always a central core of mathematical reasoning that is supposed to be logically sound. So, it is fairly reasonable and intuitive to consider multi-valued logic or fuzzy logic. So, if our conversation was to be two-valued rather than fuzzy valued, then our daily life would be more difficult. For example, if we have a red piece of cloth with a yellowish tint we may be in doubt whether we should call the color red or red- yellow, and we may even disagree about the name we wish to give to the color. Even though classical mathematics deals with concepts subject to the rules of two valued logic, in particular to the postulate of the excluded middle, it is not true that all statements involve concepts that are subject to logic. Concepts of the classical logic need to be changed to reflect and express reality more effectively. It must be realized in mathematics that reality is more or less uncertain, vague, and ambiguous. Mathematics education has a point of weakness, since it deals with infinite valued subjects in a two-valued way. It is the same as when we watch an ocean from one place on a beach. The widespread public image of mathematics is that it is difficult, cold, inhuman, abstract, theoretical, ultra-rational, and relates absolutist philosophies of mathematics. It is argued that this image is consistent with separated values. In contrast, an opposing humanized image of mathematics, consistent with connected values, finds academic support in recent fallibilist philosophies of mathematics. It is argued also that although these two philosophical positions have a major impact on the ethos of the mathematics classroom, there is no direct logical connection. It is concluded that the values realized in the classroom are probably the dominant factors in determining the learner's image and appreciation of mathematics.

Ernest's [3] conclusion mentioned above is based on an existing mathematics which is bivalent. However, a humanized image of mathematics, which is consistent with connected values, is not bivalent. The object of study in mathematics education might be, for example, the teaching of mathematics; the learning of mathematics; teaching and learning situations; the relations between teaching, learning and mathematical knowledge; the reality of mathematics classes; societal views of mathematics and its teaching; or the system of education itself. The teaching learning process is considered as a social interact

In addition, because the concept of intuition is fuzzy, it is more appropriate to use fuzzy logic with the concept of intuition than two-valued logic. To do that, mathematics education must be conducted for students, and teachers must listen to their difficulties in learning mathematics. Thus to help the students, the most important thing for teachers is to let their students use natural language in learning mathematics in a constructivist learning environment, and learn fuzzy mathematics which is consistent with social sciences.

III. Origin of Fuzzy Concepts

Plato laid the foundation of fuzzy logic, indicating that there was a third region (beyond True and False) where these opposites "tumbled about." Other, more modern philosophers echoed his sentiments, notably Hegel, Marx, and Engels. But it was Lukasiewicz [9] who first proposed a systematic alternative to the bi-valued logic of Aristotle. In the early 1900's, Lukasiewicz [9] described a three-valued logic, along with the mathematics to accompany it. The third value he proposed can best be translated as the term "possible" and he assigned it a numeric value between True and False. Eventually, he proposed an entire notation and axiomatic system from which he hoped to derive modern mathematics. Later, he explored four-valued logics, five-valued logics, and then declared that in principle there was nothing to prevent the derivation of an infinite-valued logic. Lukasiewicz [9] felt that three- and infinite-valued logics were the most intriguing, but he ultimately settled on a four-valued logic because it seemed to be the most easily adaptable to Aristotelian logic. Knuth [10] proposed a three-valued logic similar to Lukasiewicz's[9], from which he speculated that mathematics would become even more elegant than in traditional bi-valued logic. Knuth’s [10] insight, apparently missed by Lukasiewicz [9], was to use the integral range [-1, 0 +1] rather than [0, 1, 2]. Nonetheless, this alternative failed to gain acceptance, and has passed into relative obscurity. It was not until relatively recently that the notion of an infinite-valued logic took hold. In 1965 A Mathematician and Computer Scientist from Iran, Lofti Zadeh [8] published his seminal work "Fuzzy Sets" ([5], [6]) which described the mathematics of fuzzy set theory, and by extension fuzzy logic. He formalized this into Fuzzy Sets and Fuzzy Logic just like Relational Sets and Relational Math and made it very useful for a number of purposes. This is of most use in Robotics, where the robots need to make judgments and Fuzzy Logic is more useful than other systems in Mathematics. This theory proposed making the membership function (or the values False and True) operate over the range of real numbers [0.0, 1.0]. Mathematics, before this, did not help too much in handling gradations in values. It just cut off things at the required limits.

Fuzzy logic was big during the 80's and then was sort of folded into other things in the last two decades plus. It does not take away the usefulness of Fuzzy thinking! In 1963 R.H. Wilkinson has given a proposal for the term "fuzzy" as a multi-valued logic that is very useful in analog applications. He also built analog circuits that recognized gradations in some value over a range and did appropriate things depending upon the range in which it was.

IV. Need of Fuzzy Concepts

If you think being 6 feet is tall, is being 5 feet 11 inches, also tall? Things don't go from Cold to Hot in anybody's mind. There are things that are very cold, cold, cool, slightly warm, warm, very warm, hot and very hot! Those are the gradations used in Fuzzy Logic that enables you to tackle with computers! A police officer is less likely to give you a traffic ticket if the radar gun clocks you at 26 miles per hour in a 25 mile per hour zone, even if legally he or she could do so. They might if you are doing 45! However Traffic Camera at the same location is likely to send you a ticket if it catches you doing 26 mph! Fuzzy logic is like the Police Officer that uses a range of overlapping limits and uses his or her discretion in deciding whether you were speeding.

Regular Mathematics is like the traffic camera that uses a logic that has rigid limits. Sometimes it does not work for the purposes you need, especially where something approximating Human Judgment is needed [13].

You could, for example, have something have a degree of warm and at the same time have some degree of coldness. In other words, there were overlaps where a computer could be capable of discretionary judgment like a human being can. Like many other very useful research that was done in the 80's like Artificial Intelligence, Fuzzy Logic got absorbed into the background in many of the products, hardware and software that we use today. Like this Fuzzy Logic Toolkit from MatLab software. Fuzzy Logic was one of the seminal contributions to Computer Science and Robotics[13].

Although "fuzzy logic" may seem to imply imprecision, it's based on a reliable and rigorous discipline [14]. Fuzzy logic lets you accurately describe control systems in words instead of complicated math. Fuzzy logic, based on fuzzy set theory, allows you to express the operational and control laws of a system linguistically i.e. in words. Although such an approach might seem inadequate, it can actually be superior to (and much easier than) a more mathematical approach. The main strength of fuzzy set theory, a generalization of classical set theory, is that it excels in dealing with imprecision.

In classical set theory, an item is either a part of a set or not. There is no in-between; there are no partial members. For example, a cat is a member of the set of mammals, and a frog is not. Such sets are called crisp sets. Fuzzy set theory recognizes that very few crisp sets actually exist. Fuzzy logic allows partial set membership; it allows gradual transitions between fully a member of the set and fully not a member of the set. Being partially a member of a given set, a given element is also partially not a member of that set. Traditional logic recognizes only full or null membership in a set and requires that a given assertion be either true or false. Fuzzy logic, however, allows partial truth and partial falseness.

V. Fuzzy Set & Fuzzy Logic

Zedah [8] found the fuzzy logic and fuzzy set in mid sixties. In the last three decades, significant work has been done in the development of fuzzy set and fuzzy logic and their use in large number of applications. Fuzzy set and fuzzy logic are powerful mathematical tools for modeling; uncertain systems in industry, nature, the humanities; and as a facilitator for common sense reasoning in decision making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, when the goal is to find a good approximate solution. Fuzzy logic is attracting a great deal of attention in the business and industrial world as well as among the general public [11].

In mathematics, logic has vital role to play as a language and there is a correspondence between the logical connectives “and, or, not, implication” and the set of operations “intersection, union, complement, inclusion”, respectively. It is established that this correspondence (called isomorphism) guarantees that every theorem or result in set theory has a counterpart in two-valued logic and vice versa [4]. The important primary property of an ordinary set is that either an element belongs to the set or not. Ordinary logic called two-valued logic is isomorphically connected with the ordinary set. The two concepts, ordinary sets and ordinary logic, play a central role in the mathematical system.

In contrast to the stochastic uncertainty-type vagueness, the vagueness concerning the description of the semantic meaning of events, phenomena or statements is called fuzziness [12]. In fact, vagueness is no more to be done away with in the world of logic than friction in mechanics. Fuzzy set and fuzzy logic have been applied to virtually all branches of science, engineering and socio-economic sciences. All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial one. The law of the excluded middle is true when precise symbols are employed but it is not true when symbols are vague, as, in fact, all symbols are.

All languages are vague. Vagueness, clearly, is a matter of degree [4]. An important step towards dealing with vagueness was made by the philosopher Black [11] who introduced the concept of vague set.

Infinite valued logic can be further extended to fuzzy logic by incorporating fuzzy set and fuzzy relations in the system of infinite valued logic. Fuzzy logic uses as a major tool fuzzy set theory. It focuses on linguistic variables in natural language and aims to provide foundations for approximate reasoning and imprecise propositions

VI. Logic & Mathematics

The logic employed in computers is of the two valued kind, which we can represent by voltages, 0 and 1. In real computer logic chips the output changes suddenly when the input exceeds a threshold value, so we can say that all inputs between 0V and 0.5V will give one output and those between 0.5V and 1V will give the other. If we plot input voltage against output we get a step function. Many other less sharp functions are possible, including a linear function that goes smoothly from 0 to 1 in a straight line Generalizing, we can say that the step function is the most nonlinear extreme of a continuum between straight line linearity and nonlinearity. In between are many functions that have intermediate levels of nonlinearity (e.g. the sigmoid function used in neural network). Science most often deals with linear functions, so we can regard scientific formula as implementations of a sort of linear logic at one extreme, with traditional logic as a form of nonlinear mathematics at the other. Thus mathematics and logic are both equivalent and are just alternative ways of speaking about the same system - with infinite accuracy (mathematics) or just one bit accuracy (logic). The breakthrough in fuzzy logic is to recognize that there are other alternative representations in between these extremes.

We see then that the number of different values possessed by the logic can vary from just two [0, 1], up to infinity [0,...1]. Thus this wider ranging view can be called multi valued or multivalent logic, which in the limit, using all the real number values, is called fuzzy logic. The range of values available in any logic forms what is called a mathematical set. Therefore fuzzy and multi valued logic deals not with single binary numbers but with sets of numbers. These relate to both inputs and outputs, so the most general form maps input sets to output sets. To see how this works, consider the question, "which of three possible outputs (heating, cooling, off) should a air conditioner take, given temperatures in three rooms?" The extremes (all inputs lower than wanted or all higher) are easy, as is the case for all within limits, but how do we treat the case where there is one of each? Many incompatible combinations are thus possible and we need a function that can weight the relevant inputs and not treat them as just binary values.

VII. Applications of Fuzzy Concepts

Which fuzzy logic has been successfully applied are often quite concrete. The first major commercial application was in the area of cement kiln control, an operation which requires that an operator monitor four internal states of the kiln, control four sets of operations, and dynamically manage 40 or 50 "rules of thumb" about their interrelationships, all with the goal of controlling a highly complex set of chemical interactions. One such rule is "If the oxygen percentage is rather high and the free-lime and kiln-drive torque rate is normal, decrease the flow of gas and slightly reduce the fuel rate" (see Zadeh [8]). A complete accounting of this very successful system can be found in Umbers and King [7]. The objection has been raised that utilizing fuzzy systems in a dynamic control environment raises the likelihood of encountering difficult stability problems: since in control conditions the use of fuzzy systems can roughly correspond to using thresholds, there must be significant care taken to insure that oscillations do not develop in the "dead spaces" between threshold triggers. This seems to be an important area for future research. Other applications which have benefited through the use of fuzzy systems theory have been information retrieval systems, a navigation system for automatic cars, a predicative fuzzy-logic controller for automatic operation of trains, laboratory water level controllers, controllers for robot arc-welders, feature-definition controllers for robot vision, graphics controllers for automated police sketchers, and more.

Expert systems have been the most obvious recipients of the benefits of fuzzy logic, since their domain is often inherently fuzzy. Examples of expert systems with fuzzy logic central to their control are decision-support systems, financial planners, diagnostic systems for determining soybean pathology, and a meteorological expert system in China for determining areas in which to establish rubber tree orchards. Another area of application, akin to expert systems, is that of information retrieval.

VIII. Conclusion

The generalization and extension of ordinary sets and two valued logic can be easily accepted in terms of fuzzy sets and fuzzy logic respectively. Fuzzy sets and fuzzy logic lies on the cross roads of logical nature of sciences and complexities of humanities and social sciences. Being more natural and more precise they have proven to be good communication media for them.

As a mathematical system, fuzzy sets and fuzzy logic expands the current framework and build a world that takes in new concepts so they have interested researchers on the theoretical side from early on. Thus, to teach fuzzy mathematics as compared to two valued mathematics is more desirable, reasonable and advisable. Fuzzy mathematics can easily be taught by using fuzzified educational methods. Hence it is appropriate to use multi-valued fuzzy logic rather than two-valued classical mathematical logic in most of the applications.

References

[1] W. Magnus (1997): The Significance of Mathematics; The Mathematicians Share in the General Human Condition. Amer. Math. Monthly , no. 3, 261–269.
[2] L. Wittgenstein (1978): Remarks on the Foundation of Mathematics (revised ed.) MIT Press.Cambridge, Mass.
[3] P.Ernest(1989): Philosophy, mathematics and education. Int. J. Math. Educ. Sci. Technol., no.4, 555–559.
[4] Lee, Byung-Soo, Kang, Mee-Kwang Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education Vol. 1, No. 1, July 1997, 75–85
[5] L.A. Zadeh, "Fuzzy sets," Info. & Ctl., Vol. 8, 1965, pp. 338-353.
[6] L.A. Zadeh, "Fuzzy algorithms," Info. & Ctl., Vol. 12, 1968, pp. 94-102.
[7] I.G. Umbers and P.J. King, "An analysis of human decision-making in cement kiln control and the implications for automation," Int. Jrnl. of Man-Mach. Stud., Vol. 12, 1980, pp. 11-23
[8] L. A. Zadeh (1965): Fuzzy Sets, Information and Control , 338–353.
[9] C. Lejewski, "Jan Lukasiewicz," Encyclopedia of Philosophy, Vol. 5, MacMillan, NY: 1967, pp. 104-107.
[10] James F. Brule' Fuzzy Systems - A Tutorial (c) Copyright James F. Brule' 1985
[11] M. Black, Vagueness (1937): An Exercise in Logical Analysis . Philosophy of Science , 472–455.
[12] L. Moreno-Armella & G. Waldegg: Constructivism and mathematical education. Int. J. Math. Educ. Sci. Technol., no. 5(1993), 653–661.
[13] Fuzzy Logic for "Just Plain Folks" By Thomas E. Sowell, Published on the Web 2003, revised September, 2008.
[14] Brubaker, David I (1992): Fuzzy-logic basics: intuitive rules replace complex math. (includes related article that briefly describes the history and nature of fuzzy logic) Article from: EDN.