

A Mathematical Approach for Afforestation to Mitigate Climate Change

C. Vanlisuta and S. Prombanpong

Abstract—The objective of this paper is to develop a mathematical model to determine types and number of trees to plant in the forest in order to maximize profit based on such expenses as operation expenses, raw material, against revenue from carbon credit obtained from these plants. The model is, therefore subject to several constraints i.e. planting area, required amount of carbon sequestration and so on. The developed model can be used to obtain the optimal solution in the plantation system of the clean development mechanism (CDM) project. This model is currently under verification by several organizations both in private sectors and government agencies in Thailand. Therefore, for a demonstration of how to implement the model, eight different parts of Thailand are investigated and experimented. Note that only economical plants are considered in the model. As a result, it is found that solutions obtained from the optimization model are practical and acceptable. This will lead to an accurate computation of carbon credit in CDM in the future.

Index Terms—Optimization, CDM, climate change.

I. INTRODUCTION

Climate change in terms of global warming is one of the intimidating problems that threat people nowadays [1]. According to World Bank report, both CDM (Clean Development Mechanism) and JI (Joint Implementation) can reduce carbon dioxide equivalent around 832 Metric-ton [2]-[4]. In addition, reforestation is introduced in order to reduce carbon dioxide in the atmosphere. Therefore, all nations in the World attempt to preserve and replant the forests [5]. However, it is found that the reforestation process is not optimized in terms of carbon sequestration obtained from the trees. Meaning that they are planted without consideration of an expected amount of carbon absorption and costs. None research has been conducted in applying optimization in afforestation process. Let alone develops the mathematical model and apply to solve this problem. However, some of related literature papers are on different topics. Stepper developed the mathematical model to study the relationship between tree trunk diameter and its growth rate [6]. Boland developed optimal harvest time utilizing an integer programming model [7]. Jain and Clearsky developed a numerical smoke dispersion forecast system during post-harvest burning of agricultural fields [8]. Kittredge used allometric equations in a study of bio mass in the forest [9].

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The equations later were simplified to calculate bio-mass of tree trunk and branches by Satoo and Senda [10]. In order to increase accuracy in the bio mass computation, the height of the tree must be included in the calculation [11]-[13]. Alexander Clark III studied total weight of the tree and calculated carbon sequestration using diameter of the tree trunk and its height [14].

Although, an optimization technique has been used in various areas, has this technique not yet been applied much in forestation.

II. METHODOLOGY

The flow diagram of the process is divided into three steps and illustrated in Fig. 1. The first step is input. All relevant data of the mathematical model must be entered in the particular format. Next, the linear programming model is solved to obtain the optimal solution. Finally, the solution will be interpreted as in the output format in terms of selected trees and their locations, as well as total profit earned. Thus, the developed mathematical model will be described next.

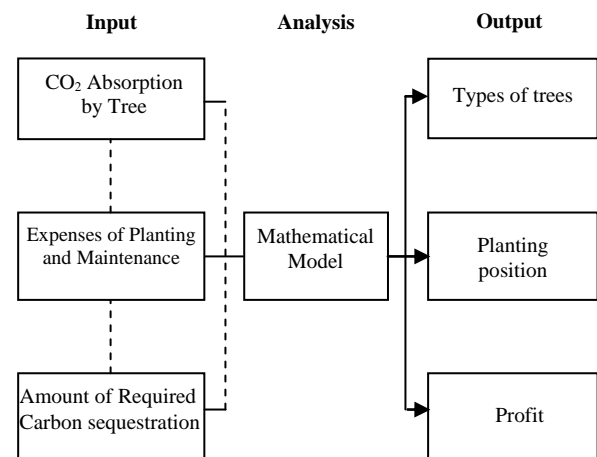


Fig. 1. Flow diagram of the model

A. Mathematical Model

This paper utilizes an integer linear programming to formulate the mathematical model. The objective function is to maximize profit which is a difference between a total return of carbon credit and the total expenses of plantation, and operation expenses [15],[16]. The objective function will be expressed as the following.

1) Objective function

$$(Max.)Z = \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K C [X_{ijk} a_i] - \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K C [X_{ijk} b_i] \quad (1)$$

where

- X_{ijk} : a tree i plants at location (j,k)
- C : a carbon credit revenue (US\$/ton carbon)
- a_i : the ability of carbon dioxide absorb of tree i (Ton carbon/year)
- b_i : expenses of planting and maintenance tree i (US\$/tree/year).
- i : tree species, where $i = 1, 2, \dots, I$
- j : row array, where $j = 1, 2, \dots, J$
- k : column array, where $k = 1, 2, \dots, K$

2) Constraints

The objective function is subjected to the following constraints.

- 1) Area constraint: The total area of planting must not greater than the available area.

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K [X_{ijk} s_i] \leq A \quad (2)$$

where s_i is the area required to plant tree i and A is a total available area.

- 2) Singularity constraint: Only one tree can be planted at only one position.

$$\sum_{i=1}^m X_{ijk} = 1; \forall j, k \quad (3)$$

- 3) Carbon sequestration constraint: The total amount of carbon sequestration by all trees must greater than the required amount of carbon absorption.

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K [X_{ijk} a_i] \geq L \quad (4)$$

where L is an amount of required carbon sequestration

- 4) Tree distribution constraint: This constraint ensures a distribution of trees throughout the area. In addition, the similar type of tree should not grow next to each other since its growth rate can be harmful.

$$X_{ijk} + X_{imn} = 1; \forall i \quad (5)$$

where $m \in \{j-1, j, j+1\}$ and $n \in \{k-1, k, k+1\}$

- 5) 0-1 Integer constraint: All decision variables are either zero or one.

$$X_{ijk} = 0,1 \quad (6)$$























B. Experimentation

Once the mathematical model is developed, the implementation will be next. The input data such as operating expenses and carbon sequestration ability of each tree must be acquired from several sources and can be shown in Table I and Table III respectively. In Table I, the expenses in the first year include seeding cost, field preparation, and labor cost [17]. The expenses for the following years until the seventh year include labor cost, fertilizing, and tree replenishment. The carbon absorption ability by a tree is in ton carbon per year which is by

courtesy of Office of Nation Resources and Environmental policy and planning, Ministry of Natural Resources and Environment [18].

In this experiment, planting area of 400 by 400 meter is utilized. The required area of each plant to be fully grown is designed at 4 by 4 meter which is considered providing the best growing result. Therefore, there will be exactly 100 trees needed to grow in the area. The required total amount of carbon sequestration is set at 0.05 ton carbon per year.

TABLE I: THE OPERATING AND MAINTENANCE EXPENSES

Legend	Tree	Expenses(\$)		Total (US\$)
		1 st year	2 nd -7 th years	
	<i>Acacia mangium</i> Willd	145	772	917
	<i>Acacia auriculaeformis</i>	178	772	950
	<i>Alstonia Scholaris</i> R. Br.	172	772	994
	<i>Anthocephalus chinensis</i> (Lam) Rich. Ex Walp.	138	772	910
	<i>Azadirachta.indica.A..Juss</i> Var. siamensis	175	772	947
	<i>Dalbergia oliveri</i> Gamble	151	772	923
	<i>Duranta erecta</i> L.	175	772	923
	<i>Duranta erecta</i> L.	168	772	947
	<i>Elaeis guineensis</i> Jacq.	150	772	940
	<i>Eucalyptus citriodora</i> .	164	772	922
	<i>Hymenocallis littoralis</i> Salosb	201	772	936
	<i>Lagerstroemia calyculata</i> Kurz	168	772	973
	<i>Mangifera indica</i> .	151	772	940
	<i>Pelltophorum pterocarpum</i>	171	772	923
	<i>Plerocapus indicus</i>	175	772	943
	<i>Plerocapus macrocarpus</i> Kurz	151	772	947
	<i>Plerocapus macrocarpus</i> Kurz	175	772	947
	<i>Shorea obtuse</i> Wall. Ex Blume	169	772	923
	<i>Shorea roxburghii</i> G. Don	235	772	947
	<i>Shorea roxburghii</i> G. Don	235	772	947
	<i>Sindora siamensis</i> Teijsm. Ex Miq.	145	772	941
	<i>Swietenia macrophylla</i> King			1,007
	<i>Tectona grandis</i> L.f.			917

The carbon credit revenue is the earning from the amount of carbon sequestration. The amount of credit earning during 2009-2012 is shown in Table II which is by courtesy of Thailand Greenhouse Gas Management Organization (Public Organization).

III. RESULTS








As mentioned earlier, the forestation area in Thailand is divided into eight different parts. This is due to the fact that each part is suitable for certain types of plants. Therefore,

the corresponding input data of each part are managed and the model is solved using LINGO software. The optimal results of each part of Thailand are shown in Table IV-Table XIII.

TABLE II: CARBON CREDIT REVENUE

Year	2009	2010	2011	2012
Carbon Credit (US\$/TCO ₂ -)	12.62	12.19	11.39	5.80

TABLE III: THE CARBON SEQUESTERED BY A TREE (TON CO₂ PER YEAR)

Legend	Tree	CO ₂ Absorption
	<i>Acacia mangium</i> Willd	126.10
	<i>Acacia auriculaeformis</i>	213.13
	<i>Alstonia Scholaris</i> R. Br.	310.81
	<i>Anthocephalus chinensis</i> (Lam) Rich. Ex Walp.	218.02
	<i>Azadirachta.indica.A..Juss</i> Var. siamensis	893.42
	<i>Dalbergia oliveri</i> Gamble	140.09
	<i>Duranta erecta</i> L.	381.86
	<i>Elaeis guineensis</i> Jacq.	45.51
	<i>Eucalyptus citriodara</i> .	273.30
	<i>Hymenocallis littoralis</i> Salosb cv	203.14
	<i>Lagerstroemia calyculata</i> Kurz	126.10
	<i>Mangifera indica</i> .	421.51
	<i>Pelltophorum pterocarpum</i>	170.95
	<i>Plerocapus indicus</i>	130.99
	<i>Plerocapus macrocarpus</i> Kurz	638.51
	<i>Shorea obtuse</i> Wall. Ex Blume	379.64
	<i>Shorea roxburghii</i> G. Don	126.10
	<i>Sindora siamensis</i> Teijsm. Ex Miq.	84.14
	<i>Swietenia macrophylla</i> King	72.82
	<i>Tectona grandis</i> L.f.	301.27

In Table IV it can be concluded that there are five kinds of trees suitable to grow in this area. They are : *Plerocapus macrocarpus* Kurz, *Acacia mangium* Willd, *Acacia auriculaeformis*, *Alstonia Scholaris* R. Br. and *Azadirachta.indica.A..Juss..Var. siamensis*. The number of trees that must be planted are 25, 20, 20, 15, 20 trees respectively. The earning of 1,739,962 \$ can be obtained within 7 years span.

In Table V it can be concluded that there are seven kinds of trees suitable to grow in this area. They are : *Plerocapus macrocarpus* Kurz, *Acacia mangium* Willd, *Acacia auriculaeformis*, *Alstonia Scholaris* R. Br, *Tectona grandis* L.f., *Tectona grandis* Linn. and *Azadirachta.Indica* A. .Juss..

Var siamensis. The number of trees that must be planted are 15, 15, 15, 10, 20, 10, 15 trees respectively. The earning of 1,803,328 \$ can be obtained within 7 years span.

TABLE IV: THE OPTIMAL SOLUTION OF EASTERN PART









































































































































































































Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

TABLE V: THE OPTIMAL SOLUTION OF WESTERN PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

In Table VI it can be concluded that there are seven kinds of trees suitable to grow in this area. They are : *Plerocapus macrocarpus* Kurz, *Mangifera indica*, *Hymenocallis littoralis* Salosb cv., *Swietenia macrophylla* King, *Elaeis guineensis* Jacq., *Hymenocallis littoralis* Salosb cv and *Shorea siamensis*. The number of trees that must be planted are 25, 15, 10, 15, 10, 10, 15 trees respectively. The earning of 1,633,536 \$ can be obtained within 7 years span.

In Table VII it can be concluded that there are seven kinds of trees suitable to grow in this area. They are : *Plerocapus macrocarpus* Kurz, *Anthocephalus chinensis* (Lam) Rich. Ex Walp., *Azadirachta.indica.A..Juss.Var siamensis*, *Swietenia macrophylla* King, *Acacia auriculaeformis*, *Acacia mangium* Willd and *Alstonia Scholaris* R.Br. The number of trees that must be planted are 10, 15, 20, 10, 15, 20, 10 trees respectively. The earning of 3,076,800 \$ can be obtained within 7 years span.

TABLE VI: THE OPTIMAL SOLUTION OF SOUTHERN PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

TABLE VII: THE OPTIMAL SOLUTION OF CENTRAL PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

TABLE VIII: THE OPTIMAL SOLUTION OF UPPER NORTHERN PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

In Table VIII it can be concluded that there are nine kinds of trees suitable to grow in this area. They are: *Sindora siamensis* Teijsm. Ex Miq., *Duranta erecta* L., *Lagerstroemia*

calyculata Kurz, *Tectona grandis* Linn., *Plerocapus macrocarpus* Kurz, *Tectona grandis* L.f., *Shorea obtuse* Wall. Ex Blume, *Shorea roxburghii* G. Don and *Xylia xylocapa*(Roxb.) Tuab.var. kerrii. The number of trees that must be planted are 10, 10, 10, 15, 15, 10, 10, 10, 10 trees respectively. The earning of 2,732,810 \$ can be obtained within 7 years span.

TABLE IX: THE OPTIMAL SOLUTION OF UPPER NORTH EAST PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

In Table IX it can be concluded that there are eleven kinds of trees suitable to grow in this area. They are : *Plerocapus macrocarpus* Kurz, *Duranta erecta* L., *Sindora siamensis* Teijsm. Ex Miq., *Tectona grandis* L.f, *Shorea obtuse* Wall. Ex Blume, *Shorea roxburghii* G. Don, *Lagerstroemia calyculata* Kurz, *Dalbergia oliveri* Gamble, *Pelltophorum pterocarpum*, *Eucalyptus citriodora* and *Xylia xylocapa*(Roxb.) Tuab.var. kerrii. The number of trees that must be planted are 10, 10, 10, 10, 5, 10, 10, 10, 5, 5, 15 trees respectively. The earning of 4,484,866 \$ can be obtained within 7 years span.

TABLE X: THE OPTIMAL SOLUTION OF LOWER NORTH EAST PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

In Table X it can be concluded that there are ten kinds of trees suitable to grow in this area. They are: *Tectona grandis* L.f., *Sindora siamensis* Teijsm. Ex Miq., *Plerocapus*

macrocarpus Kurz, Shorea obtuse Wall. Ex Blume, Shorea roxburghii G. Don, Lagerstroemia calyculata Kurz, Dalbergia oliveri Gamble, Plerocapus indicus, Xylia xylocapa(Roxb.) Tuab.var. kerrii and Eucalyptus citriodara. The number of trees that must be planted are 10, 10, 10, 10, 10, 10, 10, 10, 10 trees respectively. The earning of 4,427,323 \$ can be obtained within 7 years span.

TABLE XI: THE OPTIMAL SOLUTION OF LOWER NORTH PART

Position of tree	Type of plant									
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

In Table XI it can be concluded that there are six kinds of trees suitable to grow in this area. They are: Shorea obtuse Wall. Ex Blume, Duranta erecta L., Sindora siamensis Teijsm. Ex Miq., Tectona grandis L.f., Plerocapus macrocarpus Kurz and Toona ciliata M.Roem. The number of trees that must be planted are 20, 20, 20, 10, 20, 10 trees respectively. The earning of 4,576,608 \$ can be obtained within 7 years span.

TABLE XII: THE SUMMARY OF RESULTS

Type of plant	section			
	Eastern Part	Western Part	Southern Part	Central Part
	25	15	25	10
	20	15	0	20
	20	15	0	15
	15	10	0	10
	20	15	0	20
	0	20	0	0
	0	10	0	0
	0	0	15	0
	0	0	10	0
	0	0	15	10
	0	0	10	0
	0	0	0	15
	0	0	10	0
	0	0	15	0

Total	100	100	100	100
Profit (M\$)	1.734	1.803	1.633	3.076

TABLE XIII: THE SUMMARY OF RESULTS FOR THE REMAINING SECTIONS

Type of plant	section				
	Upper Part	Northern Part	Upper North East Part	Lower North East Part	Lower North Part
	10	10	10	20	20
	10	10	0	20	20
	10	10	10	0	0
	15	0	0	0	0
	15	10	10	10	20
	10	10	10	10	10
	10	5	10	20	20
	10	10	10	10	0
	10	15	10	10	0
	0	10	10	10	0
	0	5	0	0	0
	0	5	10	10	0
	0	0	10	10	0
	0	0	0	10	10
	0	0	0	0	10
Total	100	100	100	100	100
Profit (M\$)	2.732	4.484	4.427	4.576	4.576

IV. DISCUSSION

This paper demonstrates the utilization of the optimization technique to determine a number and type of trees to be planted in the forest in order to maximize profit. According to the result, it can be seen that the tree distribution is quite even and it is considered to be practical. This mathematical model will be one of the tools used in forestation in order to obtain required amount of carbon sequestration in the CDM project.

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