Application of Fixed Order Period Lot Sizing Rule in Capacitated Make-to-Order Production Systems

S. H. Tang, Mehdi Vasili, Mohammadreza Vasili, and N. Ismail

Abstract—**In this paper, the fixed order period (FOP) lot sizing rule is considered as an appropriate lot-sizing strategy; which, through establishing a work-ahead-window, is used for modeling make-to-order (MTO) production systems with limited available capacity. FOP in an MTO environment summarizes the known customers' demands of the next work-ahead-window periods into one production lot. Establishing a work-ahead- window helps to smooth the production and buffer the production line from demand variability by pre-production of orders that are needed later. In other words, the orders that should be delivered in periods with higher required capacity than the available capacity are pre-produced in periods with lower required capacity. Generally, the fluctuation of the customer required capacity in an MTO environment is smoothed by applying an average operator in a way that the long term average customer required capacity is always less than the available capacity.**

Index Terms—**Capacity order characteristic, fixed order period (FOP), make-to-order (MTO), work-ahead-window.**

I. Introduction

FOP lot sizing rule in a capacitated MTO production system is performed under a cyclical sequential trend in which each type of product is produced right after the production of other type. Each product is produced once only in each production cycle and for all the needs in that production cycle. The sequence of producing the products in each cycle is arbitrary but same during the planning horizon. The sequential trend in production of products in a capacitated single resource environment, also have been discussed enormously in the well-known economic lot scheduling problem (ELSP) model. As it has been stated by Elmaghraby [1], ELSP arises from the desire to accommodate the cyclical production patterns that are based on economic manufacturing quantity (EMQ) calculations for individual items on a single production facility. The production of any item first requires a setup, followed by continuous production of a specified quantity of that item. This quantity should last until the next time the facility is scheduled to produce that item [2]. Rogers [3] with a computational approach to the ELSP, firstly recommended a feasible schedule for producing economic lots of different items, while all production requirements can be met without exceeding available production capacity. He suggested, through a sequence of production periods for items arranged, all production requirements for a given time interval can be fulfilled without exceeding available production capacity at any point. In addition, it has been mentioned that by repeating the sequence the production requirements can be met indefinitely without exceeding capacity limits. However, the demand per unit time period for each item has been considered as a constant amount. Since, traditionally, ELSP model assumes that the items face constant demand at item-specific rates over an infinite time horizon, the established sequential cyclical trend for production of items in this model cannot be applied in an MTO environment. This is because; in MTO environment usually we face a considerable fluctuation in demand per-period for each item. Therefore, production period for each item should vary time by time to be able to cope with the natural fluctuation of demands in an MTO environment.

Hopp and Spearman [4] and Ohno [5], stated that, the basic connection between production and demand, in pull system at the strategic level may have close interpretation to the concept of MTO production strategy; but it is different with concept of pull system in tactical level. However, Ohno [5] has outlined the importance of leveling the production in strategic pull, that can be accomplished through establishing takt time or takt-paced production. According to Hopp and Spearman [4], takt time by setting a pace on production of products instead of chasing the demand, determine the rate of assembly or production in a factory in a way that set the output of the plant to be equal to demand. This is in order to smooth the demand variability, in an environment similar to the MTO environment, by buffering strategies such as backlog or inventory. If demand temporarily increases, orders are backlogged. If orders are needed later, the line will build up some inventory. Jodlbauer and Altendorfer [6] specifically applied FOP lot-sizing rule as a buffering strategy in the model presented by Jodlbauer [7] for a single resource capacitated multi-product production environment. This paper more specifically elaborate the concept of FOP lot-sizing rule, based on the model presented by the Jodlbauer and Altendorfer [6].

II. Methodology

A single resource multi-product capacitated production environment where products, on an MTO basis, are produced only for known and expected customers' orders is investigated. The single resource concerned may be a bottleneck or a combination of similar machine which perform similar activities, or an assembly or production line

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in a plant. In this study, in a same way as it has been expressed by Jodlbauer and Altendorfer [6], an n-periods capacity oriented work-ahead-window is introduced for managing a pre-production strategy to cope with the fluctuation of the orders in near future. This n-periods work-ahead-window is developed according to its relations with the available capacity, orders rate and capacity needed for producing them, fluctuation of the orders, and the pre-defined service-level for fulfilling the received orders. Applying the same approach as in Jodlbauer [7], by evaluating the customers buying behavior (e.g., their requested delivery lead-time) and through combining the orders required capacity with the parameters related to the delivery lead-time, production of the customers' orders which have long requested delivery lead-time with far due dates in future is relaxed. This is through extending the capacity oriented work-ahead-window to a longer work-ahead-window which also takes the unknown but expected orders in the near future into account. Therefore, only those orders which the remaining time until their due dates is shorter than the new generated work-ahead-window are released to the production process. A functional policy for managing the unknown but expected orders also is offered. The defined service-level is the probability that the long term average required capacity, for producing the final ordered products on time, should be less than the available capacity. Obviously, this service-level is determined close to 1. The demand for each product has been considered to be independently and identically distributed over time. Meanwhile, demands of different products are statistically independent.

In continue, the production of different items in a single resource capacitated MTO production environment is modeled to be able to investigate the concept of FOP lot-sizing rule mathematically. The capacity order characteristic curve and its combination by the customer required capacity to determine the system work-ahead-window also is presented in this section.

A. Required Capacity for Production of Ordered Products

At first stage, the random variable of the customers' required capacity to fulfill the orders due in period t should be defined. This well be developed according to the model of Jodlbauer and Altendorfer [6]. This random variable is defined according to its dependence on the sales amount $x_{i.t}$, processing time for production of each product of ith type p_i, setup time a_i, and the applied lot-size q_i of the product type i as follow:

$$k_t = \sum_{i=1}^{I} x_{i,t}\left(p_i + \frac{a_i}{q_i}\right), \tag{1}$$

where I denotes the total number of different types of the final products.

Generally, the customer required capacity, because of its dependence on the amount of demand for products, usually has high short term variability. Hence, in this study, the demand peaks of known customers' orders in future is manage by pre-production of these orders on stock in periods with lower customer required capacity. The required capacity for unknown but expected orders in short future also has been considered in the model of Jodlbauer [7] which will be discussed later. In order to smooth this considerable variation in customer required capacity and model the pre-production strategy, to be able to fulfill the orders in periods with requested capacity more than the available capacity, a time average operator is applied. The number of periods (n) used for this time average operator is known as capacity oriented work-ahead-window. However, the random variable of the n-periods average of customers' required capacity with due date in period t during the n-periods of the capacity oriented work-ahead-window is defined as follow:

$$k_{t,n} = \sum_{i=1}^{I}\left(p_i + \frac{a_i}{q_i}\right)\frac{1}{n}\sum_{\tau=t}^{t+n-1} x_{i,\tau} \underset{\frac{1}{n}\sum_{\tau=t}^{t+n-1} x_{i,\tau} = \overline{x_{i,\tau}}\ \&\ \sum_{\tau=t}^{t+n-1} x_{i,\tau} = q_i}{=} \sum_{i=1}^{I}\left(p_i\overline{x_{i,\tau}} + \frac{a_i}{n}\right) \underset{\sum_{i=1}^{I} a_i = A}{=} \sum_{i=1}^{I} p_i\overline{x_{i,\tau}} + \frac{A}{n}, \tag{2}$$

where; $\frac{1}{n}\sum_{\tau=t}^{t+n-1} x_{i,\tau}$ or $\overline{x_{i,\tau}}$ is the random variable of n-period average of the sales rate for all the ordered final products type $i$which should be delivered each period during work-ahead-window (when $x_{i,\tau}$ is assumed to be independently and identically distributed for each product over time). The demands of different final product types are also statistically independent. The q_i also through applying FOP lot-sizing rule is equal to $\sum_{\tau=t}^{t+n-1} x_{i,\tau}$ while the duration of the period is assumed to be one pre-defined time unit, for example one day. The summation of all change-over time is denoted by A.

If the $k_{t,n}$ is less than the available capacity and all known future customers' orders are released to production n (work-ahead-window) periods before the requested due date, then all the expectations of the customers about the delivery lead-time of the known orders will be met. Note that, a random variable that is created by the average of a sample of n identical and independent distributed random variables has the mean equal to the mean of each of the individual random variables of that sample. Meanwhile, it has the variance equal to the variance of each of the individual random variables of that sample divided by sample size (n). Through applying this fact, the initial factors for determining the adequate number of n (work-ahead-window) periods can be created accordingly. This will happen in a sufficient manner to maintain the pre-defined service-level, for satisfying the known orders, based on the available capacity, on time. However, the mean and variance of random variable n-period average customers' require capacity $k_{t,n}$ are defined as follow:

$$\mu_{k,n} = \sum_{i=1}^{I} p_i \mu_{x_i} + \frac{A}{n} = \underset{\sum_{i=1}^{I} p_i \mu_{x_i} = \mu_k}{=} \mu_k + \frac{A}{n}, \tag{3}$$

and

$$\sigma_{k,n}^2 = \frac{1}{n}\sum_{i=1}^{I} p_i^2 \sigma_{x_i}^2 \underset{\sum_{i=1}^{I} p_i^2 \sigma_{x_i}^2 = \sigma_k^2}{=} \frac{1}{n}\sigma_k^2 \tag{4}$$

where μ_{x_i} and $\sigma^2_{x_i}$ are the expectation value and the variance of the sales of product type i in one period, respectively.

As it has been suggested by Jodlbauer [7] and Jodlbauer and Altendorfer [6], the n-periods average of customers' required capacity with the mean of $\mu_{k,n}$ and variance of $\sigma^2_{k,n}$ is modeled through applying the normal distribution characteristics. In continue, an equation is offered for calculating the number of n-periods (capacity oriented work-ahead-window) needed for modeling the pre-production strategy of products, to maintain the capacity oriented service-level θ with available capacity of K, in a same way as model of Jodlbauer and Altendorfer [6]. In order to prevent a continuous increasing in the number of backorders, without loss of generality, it is assumed that $\mu_{k,n}$ is less than or equal to available capacity K. Hence, at this stage only the right tail of the distribution density function is taken into the account to consider the criterion related to available capacity appropriately. Meanwhile, the service-level, related to the delivery of products, is determined near 1.

If $F_{k.n}(K) = \theta$, then $F_{N(0,1)}\left(\frac{K-\mu_{k,n}}{\sigma_{k,n}}\right) = \theta$; thus the following equation can be solved, according to the available capacity and other parameters related to the customers' required capacity for production of final products, to find the adequate number of n periods needed, while $\mu_{k,n} \leq K$.

$$\frac{K - \mu_k - \frac{A}{n}}{\frac{1}{\sqrt{n}}\sigma_k} = F^{-1}_{N(0,1)}(\theta),\ so,\ n(k - \mu_k) - \sqrt{n}F^{-1}_{N(0,1)}(\theta)\sigma_k - A = 0,$$

hence,

$$n = \left(\frac{F^{-1}_{N(0,1)}(\theta)\sigma_k + \sqrt{\left(F^{-1}_{N(0,1)}(\theta)\sigma_k\right)^2 + 4(K - \mu_k)A}}{2(K - \mu_k)}\right)^2 \quad (5)$$

where $F^{-1}_{N(0,1)}(.)$ represents the quantile, according to the inverse of standard normal distribution function.

However, the capacity oriented work-ahead-window that equals the duration of the time periods determined for managing the pre-production strategy of known orders; or the time periods used for averaging is defined by,

$$h_K = n \quad (6)$$

B. Capacity order Characteristic

At the second stage, in a same way as model of Jodlbauer and Altendorfer [6], through consideration of the customer's requested delivery lead-time as well as the customer's buying behavior, the capacity order characteristic is developed. Capacity order characteristic answers the question of how much of the required capacity of each of the customers' orders is booked how long before the customer's requested due date. The statistical distribution of capacity delivery lead-time can be determined based on the statistical distribution of the customers' requested delivery lead-time for different product types. The random required delivery lead-time of each individual customer for the i^{th}product type L_i (as a continuous random variable) is defined as the time period between receiving each of the customers' orders for the product type$i$ and the customer's required due dates with mean of μ_{L_i} and variance of $\sigma^2_{L_i}$. Hence, the random variable of the required capacity delivery lead-time $W(n)$ is defined as the workload weighted average of L_is for all product types. The mentioned workload here refers to processing as well as the setup time for production of products according to the applied FOP lot-sizing rule.

$$W(n) = \frac{1}{\sum_{i=1}^{I}\left(p_i\mu_{x_i} + \frac{a_i}{n}\right)}\sum_{i=1}^{I} L_i\left(p_i\mu_{x_i} + \frac{a_i}{n}\right) \quad (7)$$

where $W(n)$ represents the time period between when the required capacity is booked by each of the customers' orders and the customer's required due date.

The mean and variance of the $W(n)$ are defined as follow:

$$\mu_{W(n)} = \frac{1}{\sum_{i=1}^{I}\left(p_i\mu_{x_i} + \frac{a_i}{n}\right)}\sum_{i=1}^{I} \mu_{L_i}\left(p_i\mu_{x_i} + \frac{a_i}{n}\right) \quad (8)$$

$$\sigma^2_{W(n)} = \frac{1}{\sum_{i=1}^{I}\left(p_i\mu_{x_i} + \frac{a_i}{n}\right)^2}\sum_{i=1}^{I} \sigma^2_{L_i}\left(p_i\mu_{x_i} + \frac{a_i}{n}\right)^2 \quad (9)$$

In addition, the capacity order characteristic $(O_n(t))$ is defined as 1 minus the cumulative distribution function of capacity delivery lead-time toward the delivery lead-time and the remaining time to the due date requested by the customer.

$$O_n(t) = 1 - F_{W(n)}(t) \quad (10)$$

The capacity order characteristic $O_n(t)$ indicates how much of the required capacity for production of final products is booked how long before the customers' requested due dates.

C. Combination of required capacity of ordered products and the capacity order characteristic

At the third stage, in a same way as model of Jodlbauer and Altendorfer [6], customer required capacity and the capacity order characteristic are combined. This combination is illustrated in Fig. 1.

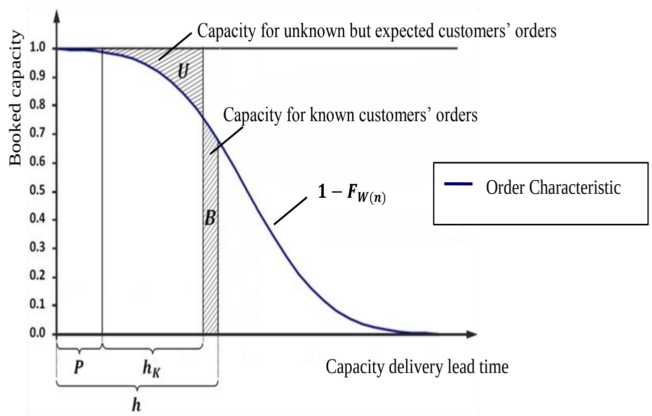

Fig. 1. Combination of required capacity of ordered products and the capacity order characteristic (Modified from Jodlbauer [7]; Jodlbauer and Altendorfer [6])

In order to run this customer driven production planning model, the minimum possible delivery lead-time of final products $P$is defined as the sum of all required change over time, processing time related to the production of products, and some other common waiting or transportation time during the production process to finalize at least one of the largest expected orders and deliver it to the customer. P does not include: waiting or queuing time in front of the single production stage, the delay causes by inventory of final products, and also the time required for providing the component parts.

In Fig. 1 the area U represents the proportion of capacity which is allocated to unknown but expected customers' orders with requested due date in near future within $P + h_K$. According to the (6), h_K is equal to n. Since U is the area allocated to the unknown orders, the product types that related to these orders are unknown too. To avoid an anonymous production of unknown orders to stock; and get free capacity for short term orders, products which are well defined by known customer orders but with due dates farther in future are pre-produced. The required capacity for this pre-production is illustrated by the area$B$.

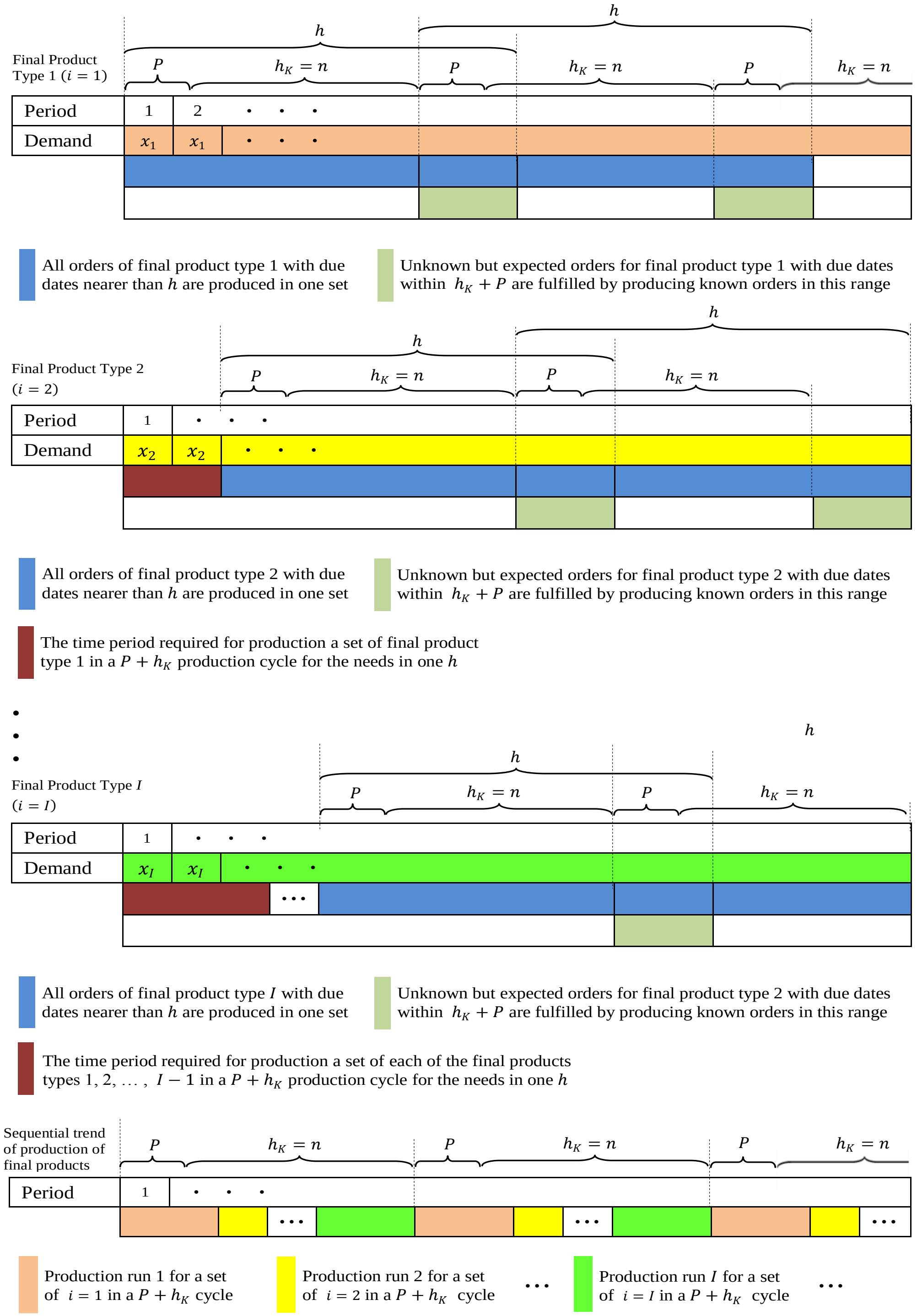

Fig. 2. Production trend of final products based on the applied FOP lot-sizing rule

In order to manage all expected short term customers' orders, with close attention to the system restrictions and assumptions, the area U should be equal to the area B (proportion of capacity needed by known customers' orders for final products with requested due dates in the further future). Then, the system work-ahead-window h for production of products, which is defiantly longer than the capacity oriented work-ahead- window h_K,is determined by the condition U should be equal to B. If all known customers' orders of final products which have a requested due date nearer than h are released to production, then all the requirements about the delivery of products for all known and expected orders, according to the determined service-level θ, will be obtained. In Jodlbauer and Altendorfers' [6] model, it also has been mathematically proved that, in order to be able to deliver all the known and expected orders on time, with respect to the available capacity, $\mu_{W(n)}$ should always be greater than the $P + h_K$.

Generally speaking, taking into the account of all these factors has tempted us that in Fig. 2 illustrate the trend in which products are produced, based on the discussed FOP lot-sizing rule, in an MTO capacitated single resource multi products production environment.

In Fig. 2, $x_i s$ are random variables which represent the amount of products which should be sold or delivered in each period during the planning horizon. According to the FOP applied lot-sizing rule, all the orders of a system work-ahead-window is summarized into one production lot. The system work-ahead-window consist of minimum possible delivery lead time P, capacity oriented work-ahead-window h_K, and a period of time that is required for producing known orders with requested due-dates in further future to fulfill unknown but expected orders in near future with due dates during$P + h_K$. A production cycle is equal to a $P + h_K$ period length and all kinds of products are produced in a sequential trend in a way that each kind of product is produced right after the production of other kind. The sequence in production of different kinds of products in each production cycle is arbitrary but same during the planning horizon. Each kind of product is produced once only during each production cycle and for all needs related to known and expected orders in a production cycle. While this sequence is repeated in different production cycle during the planning horizon and $\mu_{k,n} \leq K$and $\mu_{W(n)} > P + h_K$, all the needs can be fulfilled with respect to the available capacity and the pre-defined service-level.

III. Conclusion

In this study, application of FOP lot-sizing rule investigated in a capacitated MTO single resource multi product production environment. It was shown that through establishing a capacity orientedwork-ahead-window, fluctuation of the customers' orders can be smoothed in a way that long term average customers' required capacity is always less than the available capacity. Also, it was explained that, FOP in an MTO environment summarizes the known customers' demand of a work-ahead-window periods into one production lot. In addition, in a capacitated MTO environment products should follow a sequential cyclical production trend to be able to be produced on time based on the predefined service level. In this study, the available capacity for production of products had been considered as a constant amount during the entire planning horizon. As an extension the obtained results of this study can be investigated in a production system where there is a possibility to hire seasonal available capacity on some occasions during the planning horizon.

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