

Perishable Inventory Management and Dynamic Pricing using TTI Technologies

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Abstract—This work considers a computer simulation model for the inventory management of perishable products. Typical examples of such products are food, beverage, and pharmaceuticals. In order to keep track of the age and quality of such units in stock, time-temperature indicator (TTI) technologies can be used. The problem is to maximize the retailer's expected profit in the presence of TTIs with customers' satisfaction being taken into account. The sensing technology and information given to customers is tested in three modes: (i) a basic mode where TTI is not available; (ii) a mode in which TTIs are connected to each item and on-line alerts are made whenever the item is damaged, and (iii) a case when TTIs have the ability to predict the real expiry date with some prediction error. The model is constructed for stochastic data. Results of the simulation are reported.

Index Terms—Perishable products, inventory management, dynamic pricing, TTI technology applications.

I. INTRODUCTION

This paper considers a computer-aided simulation model for the inventory management of perishable products monitored by time-temperature indicator (TTI) tags. Typical examples of such products are food, beverage, pharmaceuticals, and human blood. Usually each perishable item in inventory is given a label that denotes its *expiry date*, after which the item is formally not proper for the original usage. In order to keep track of age and quality of such units in stock, RFID-supported smart tags can be used.

In recent years within the realm of inventory management, a technology called TTI was developed. This technology enables firms to determine on-line the actual quality situations of product types with respect to their expiry dates.

Since unexpected events may cause inventory items to be damaged before their expiry dates, the damaged goods might inadvertently be sold, which harms performance, sometimes significantly, unless a TTI-based automatic device (AD) is incorporated. An outcome of the technology is also a reduction of the risk of selling damaged products (before their expiry dates).

There are a large number of time-temperature indicators, based on different technologies. Simple devices are based on migration of dye through a filter paper while more complicated ones contain pouches with bacterial fluids that change colour when a certain time-temperature combination is reached.

Sophisticated TTI devices combine chemical and

biological functionality with RFID technology ([2], [10]). So they can remotely warn users and automated systems about spoilage whenever needed. The labeling system may include electronic circuitry that measures, calculates, and emits a signal for discounted sale. Sophisticated TTIs may have the ability to predict the real expiry date with some prediction error.

A possible way to motivate customers to purchase items and increase the expected profit is the usage of dynamic pricing, e.g., offering a discount that increases when the expiry date approaches. This policy of price differentiation within the framework of perishable or deteriorating inventory has been discussed in [11], [8], [9], and [1]. A considerable amount of work has been done on the analysis of perishing inventory systems (see, e.g., [4], [5], [3], [7], and [6], among many others). However, as of now, no published work is known to us on the modelling and simulation of perishable inventory with dynamic pricing when TTI technology is incorporated.

In this paper we study the economic effect of incorporating TTI in a perishable inventory system on the effectiveness of price differentiation policies. The sensing technology and information given to customers is tested in three modes: (i) a basic model where TTI is not available; (ii) a model in which TTIs are connected to each item and on-line alerts are made whenever the item is damaged, and (iii) a case when TTIs have the ability to predict the real expiry date with some prediction error.

The modelling frame is introduced in Section II. The stochastic nature of the inventory system is detailed in Section III. Experimental design and results are discussed in Section IV, while Section V concludes the paper.

II. DESCRIPTION OF THE MODEL

The inventory system under study consists of the following main components: a manufacturer, a retailer, customers, and a perishable product. The behavior of the inventory system is characterized by three time-varying processes.

The first is the *Demand Process*, which defines consumer requests for the product over time. The second is the *Spoilage Process*, which defines the dynamics of the product life duration. The third one is the *Selection Process*, through which an arriving consumer selects a particular item in inventory, depending on its price and degree of freshness.

The paper is concerned with the problem of periodic perishable inventory scheduling with dynamic pricing. A periodic review policy is assumed to replenish inventory up to level I_0 every cycle (period) of T units of time.

Scheduling the system involves specifying the timing (i.e., the T value) and quantity of shipments I_0 to the inventory system. In this problem customer demands are assumed to

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be stochastic and dependent upon the item's utility level, which, in turn, depends on item's price and freshness.

The objective is to maximize an expected profit for a unit of time which is composed of the revenues of the sold items minus the ordering and holding costs and penalties for the goodwill loss. This study investigates how the expected profit of the firm depends on the pricing policy, and on other factors, such as cost of applying the AD, variability of time until an item becomes defective, etc.; the main attention being devoted to the question of how the decision to include or not to include TTIs in the inventory affects the expected profit.

A. Model and Assumptions

We model the stochastic customer demands assuming that a customer enter the system randomly and either buys or does not buy an item depending on the item's utility level and its characteristics. For simplicity, we define the utility level $U^n(p, \delta)$ gained by entering customer n for a certain item as an additive function $U^n(p, \delta) = A - p + \alpha\delta$, where A, α are function parameters, p is price, and δ is freshness measured in time units until expiration. Parameter A defines the utility level gained when the item has expired (i.e., $\delta = 0$) and has no price. Parameter α defines the marginal increase in utility for having a product that is fresher by a unit time (at a fixed price). The utility decreases whenever p grows and/or δ decreases.

For simplicity, A, p and $\alpha\delta$ are normalized and measured in monetary (profit) units. Such a modelling scheme enables each customer to withdraw his current demand request, even for fresh items, if the utility gained by the customer does not exceed a pre-specified value U_{\min}^n . By constructing this utility function, we assume that the policy of reducing price increases demand as compared with a fixed price.

Events of periodic inventory replenishments are denoted by T_m , where $T_m = (m-1)T$ for $m = 1, 2, \dots, \infty$. At any time instant t , the entire set of product items $\{Z_t\}$ in inventory is divided into $N(t)$ mutually disjoint product subsets $\{Z_t^k\}$, $k = 1, 2, \dots, N(t)$, each being characterized by different arriving times to the inventory system and expiry dates, where $\{Z_t\} = \bigcup_{k=1}^{N(t)} \{Z_t^k\}$.

B. Notation

The notation includes decision variables, indices, and problem parameters.

1) Decision variables

T - planning cycle length

I_0 - order quantity

β - price differentiation intensity (to be defined later)

AD - type of automatic detection (AD) TTI to be used.

2) Indices

t - time index

i - items' index ordered according to their arrival instants

$i = 1, 2, \dots, \infty$

n - customer index, $n = 1, 2, \dots, \infty$

k - product subset index, sequenced in descending order of freshness, $k = 1, 2, \dots, N(t)$

m - index of inventory replenishment instants, where

$$T_m = (m-1)T \text{ for } m = 1, 2, \dots, \infty$$

In what follows, we distinguish between *constant parameters* appearing in problem formulation (most of which are controlled in simulation), *varying*, or *depending parameters* (which are defined through the constant ones), and *stochastic parameters*.

3) Constant problem parameters

c - cost of purchasing a single item

K - fixed ordering cost

h - cost of holding an item in storage for a unit time

A - reference level of utility for a priceless and expired item.

E - duration of time that starts at the arrival of a shipment and ends at its assigned expiry date,

p_0 - price assigned to a fresh item of level E ,

c^{AD} - additional item cost for applying AD technology,

η - penalty cost for selling a defective item,

$\hat{\pi}$ - cost of loss of goodwill per unit of time in shortage,

c^{out} - cost of removing a damaged item,

α_n - attitude of the n -th customer towards freshness

when compared to price,

U_{\min}^n - minimum utility gained by the n -th entering customer,

γ - error of product expiry prediction,

$1/\mu$ - expected time between successive demand events,

$1/\mu_D$ - expected time until the item becomes defective,

σ_A - variability in density function of reference level of utility for a priceless and expired item,

σ_d - variability in density function of time between demand events,

σ_D - variability in density function of time until an item becomes defective.

4) Varying problem parameters

T_m - inventory replenishments events, $m = 1, 2, \dots, \infty$.

E_m - expiry date of arriving shipment m , where

$$E_m = T_m + E \text{ for } m = 1, 2, \dots, \infty$$

$p_{i,t}$ - price of item i at time t ,

$\delta_{i,t}$ - freshness level of item i at time t ,

$U^n(p_{i,t}, \delta_{i,t})$ - utility gained by customer n if he purchases item i at time t

5) Random parameters used in simulation

τ_n - time points of demand, $n = 1, 2, \dots, \infty$,

τ_i^{Defect} - a specific point of time at which the i -th item will become defective,

τ_i^{pred} - a specific point of time at which the i -th item is predicted to become defective,

$S_n^{(1)}$ - a chosen item ($= 0$ if no choice is made) by an incoming customer of index n ,

$S_n^{(2)}$ - quality of the chosen item, i.e., a good or defective item (= 1 if good, and 0 if defective), by an incoming customer of index n ,

$N(t)$ - number of subsets in the inventory system at time t ,

$R(t)$ - the oldest subset type (among indices $k = 1, 2, \dots, N(t)$) accessible in inventory at time t ,

P_t - accumulated number of defective items that were eventually purchased by consumers by time t ,

D_t - accumulated damaged items (including expired items) by time t ,

$I_{k,t}^d$ - inventory level, which also includes defective items (subscript d indicates “defective item”) of product subset k at time t ,

$I_{k,t}$ - non-defective inventory level of product subset k at time t .

C. Mathematical Formulation

The suggested inventory policy is to differentiate prices among inventory items i , $i = 1, 2, \dots, \infty$, according to the freshness time $\delta_{i,t}$ considered by customers. Item i arriving in shipment m at time T_m is assigned a decreasing transient price function $P_{t,i}$ in order to reflect a decrease in price with the decreasing time interval left until the expiry date.

The mathematical formulation of the perishable inventory scheduling problem is as follows:

$$\delta_{i,t} = E + \frac{1}{\beta} \ln \left(\frac{P_{i,t}}{p_0} \right), \forall t, \forall i \in \{Z_i\}$$

$$U^n(p_{i,t}, \delta_{i,t}) = A - p_{i,t} + \alpha_n \delta_{i,t}, \forall t, \forall i \in \{Z_i\}, \forall n = 1, 2, \dots, \infty$$

$$S_n^{(1)} = \arg \max_i \left\{ \theta(U^n(p_{i,\tau_n}, \delta_{i,\tau_n}) - U_{\min}^n(p_{i,\tau_n}, \delta_{i,\tau_n})) \right\}, \forall n = 1, 2, \dots, \infty$$

$$S_n^{(2)} = \theta(S_n^{(1)}) \theta(\tau_{S_n^{(1)}}^{\text{Defect}} - \tau_n), \forall n = 1, 2, \dots, \infty$$

$$P_t = \sum_{n=1}^{\infty} \theta(S_n^{(1)}) [1 - \theta(S_n^{(2)})] \theta(t - \tau_n), \forall t$$

$$D_t = P_t + I_t^d - I_t + \sum_{m=1}^{\infty} I_{R(E_m), E_m}^d \theta(t - E_m), \forall t$$

$$I_t = \sum_{k=1}^{N(t)} I_{k,t}, \forall t$$

$$I_t^d = \sum_{k=1}^{N(t)} I_{k,t}^d, \forall t$$

$$I_{k,t}^d = |Z_i^k|, \forall k = 1, 2, \dots, N(t), \forall t$$

$$I_{k,t} = \sum_{i \in |Z_i^k|} \theta(\tau_i^{\text{Defect}} - t), \forall k = 1, 2, \dots, N(t), \forall t$$

$$I_0 \geq 0, T \geq 0, \beta \geq 0, I_{k,t} \geq 0, \forall k, t$$

Maximize the expected profit $f(I_0, T, p)$

(which depends on whether or not TTIs are used)

subject to

In order to model price differentiation offered by the firm, we assume a general exponential relation

$$\delta_{i,t} = E + \frac{1}{\beta} \ln \left(\frac{P_{i,t}}{p_0} \right) \quad (1)$$

Between price $p_{i,t}$ and freshness $\delta_{i,t}$, where p_0 is the price assigned to a fresh item of level E and parameter β corresponds to the intensity that price decreases with unit decrease in freshness.

This expression assures that for fresh items (as determined by the manufacturer) $\delta_{i,t} = E$ and $p_{i,t} = p_0$. We model the use of TTI devices for on-line automatic detection (AD) of spoiled products. It is assumed that c, h and K are independent of the quantity ordered. In situations of shortage, the system is empty and does not supply the product.

The status of the discussed inventory system at any time t is denoted by the total inventory level $[I_{1,t}^d, I_{2,t}^d, \dots, I_{N(t),t}^d]$, which also includes defective items (subscript d denotes “defective”). The total inventory, as a result, is $I_t^d = \sum_{k=1}^{N(t)} I_{k,t}^d$, where we define $I_{1,0}^d = 0$. The status of non-defective inventory at any time t is denoted by the total inventory level $[I_{1,t}, I_{2,t}, \dots, I_{N(t),t}]$. The total

non-defective inventory is $I_t = \sum_{k=1}^{N(t)} I_{k,t}$, where $I_{1,0} = 0$.

On replenishment, the status vector is computed just before the replenishment instant.

The sensing technology and information given to customers is tested in three modes: (i) a basic mode where TTI is not available; (ii) a mode in which TTIs are connected to each item and on-line alerts are made whenever the item is damaged, and (iii) a case when TTIs have the ability to predict the real expiry date with some prediction error. The analysis is separated into three different cases as follows.

Without AD: As items might be damaged by unexpected events, which customers are unaware of, these defective items might also be purchased. Customers will only later become aware of the defects. We model the compensation penalty cost for each defective item that is eventually purchased by η , $\eta \geq c$. As defective items are not detected on-line, they are removed only on expiry or purchased at request moments of demand τ_n , $n = 1, 2, \dots, \infty$. Therefore, process I_t^d is scheduled at time points of demand τ_n , $n = 1, 2, \dots, \infty$, at instants of inventory replenishments T_m , where $T_m = (m - 1)T$ for $m = 1, 2, \dots, \infty$, and at instants of expiry E_m , $m = 1, 2, \dots, \infty$, while process P_t is scheduled at a subset of time points τ_n , $n = 1, 2, \dots, \infty$.

The expected profit function $f(I_0, T, p)$ to the firm for a unit of time is expressed in the following form:

$$f(I_0, T, p) = \lim_{M \rightarrow \infty} \frac{1}{MT} \sum_{m=1}^M E \left[\sum_{n=1}^{\infty} p_{S_{\tau_n}^1, \tau_n} \theta(I_{S_{\tau_n}^1, \tau_n}^d) \theta(S_{\tau_n}^1) \theta(T_{m+1} - \tau_n) \theta(\tau_n - T_m) + \left[K \theta(I_0 - I_{T_m}^d) + c(I_0 - I_{T_m}^d) + c^{out} I_{R(E_m), E_m}^d + h \int_{T_m}^{T_{m+1}} (I_t^d) dt + \hat{\pi} \int_{T_m}^{T_{m+1}} \text{Sign}(I_t^d) dt + \eta(P_{T_{m+1}} - P_{T_m}) \right] \right] \quad (2)$$

where $\theta(x)$ is a step function equal to 1 if $x > 0$ and 0 otherwise; the function $\text{Sign}(x)$ is defined as follows: $\text{Sign}(x) = 1 - \theta(x)$, and ω denotes a possible scenario over time. By definition, $I_0^d = 0$ and $R(t)$ represents the oldest subset type accessible in inventory at time t . On expiry, $R(t)$ is computed just before the instant of expiry. The value of $R(t)$ can be increased only at replenishment instants. Furthermore, $I_t^d(\omega)$ includes defective items and does not include the expired ones. We assume that the expired items are removed immediately.

With detecting AD: When applying an AD, we assume that damaged items are on-line detected and drawn from the system when a spoilage event occurs. Items in shipment m at the expiry date E_m , $m = 1, 2, \dots, \infty$, that are still in inventory are being removed immediately from inventory, each with cost c^{out} . The cost of initially purchasing AD is modelled by an additional direct cost of c^{AD} for each item.

The expected profit function $f_{AD}(I_0, T, p)$ is:

$$f_{AD}(I_0, T, p) = \lim_{M \rightarrow \infty} \frac{1}{MT} \sum_{m=1}^M E \left[\sum_{n=1}^{\infty} p_{S_{\tau_n}^1, \tau_n} \theta(I_{S_{\tau_n}^1, \tau_n}^d) \theta(S_{\tau_n}^1) \theta(T_{m+1} - \tau_n) \theta(\tau_n - T_m) + \left[K \theta(I_0 - I_{T_m}) + (c + c^{AD})(I_0 - I_{T_m}) + c^{out}(D_{T_{m+1}} - D_{T_m}) + h \int_{T_m}^{T_{m+1}} (I_t) dt + \hat{\pi} \int_{T_m}^{T_{m+1}} \text{Sign}(I_t) dt \right] \right] \quad (3)$$

With predicting AD: In this case all the customers are on-line informed of the actual predicted expiry times, in addition to the automatic alert for removing damaged items as in the detecting AD case. Since the inventory system operates exactly the same as the inventory system with AD informing only about the quality status, the performance measure describing the expected profit remains the same. However, since additional information is given to customers, the dynamic price offered for each item should take it into account. Specifically, the interval of time $\delta_{i,t}$ affecting the price is given by $\delta_{i,t} = \min(E_m, \tau_i^{pred})$.

III. THE BEHAVIOR OF THE MODEL

The optimization problem described above is a stochastic non-linear mathematical programming problem, for analysis of

which we suggest a simulation model. The system behavior is modeled by several sources of uncertainty. The first is the *Demand Process*, which schedules the exact timing of a consumer request for the product. The demand rate is influenced by the freshness of the product (and price) as it is understood by entering customers. In order to consider this effect, we allow each arriving customer to independently react (by buying or rejecting an item) to the information about price and freshness that is available to him.

The demand process is described by an initial value of inventory level I_0 and by sequence of future jumps $\tau_0 \leq 0 < \tau_1 < \tau_2 < \dots$, which are generated by time-independent random variables of successive inter-arrival times $\tau_n - \tau_{n-1}$, denoted here by ϕ_n^{Demand} for $n = 1, 2, \dots, \infty$ and are assumed being identically distributed with expected time of $1/\mu$.

The next source of uncertainty is the *Spoilage Process*, which schedules events that cause items in inventory to become randomly spoiled. To model this process, arriving items are sequenced by an ascending index $i = 1, 2, \dots, \infty$. Each item i that enters the inventory system is assigned a specific future point of time τ_i^{Defect} at which it will become defective.

The interval of time from the arrival of item i to the system at time t until it becomes defective is denoted by ϕ_i^{Defect} , i.e., $\phi_i^{Defect} = \tau_i^{Defect} - t$.

Variables ϕ_i^{Defect} are assumed being identically distributed for each entering item i , $i = 1, 2, \dots, \infty$ with expected time of $1/\mu_D$. An Erlang probability density

function of variables ϕ_i^{Defect} is used in our simulation study in order to allow items that stay longer on the shelf a higher probability of being defective.

In the considered modes (either without or with TTI), the duration until the off-line expiry date that is determined by the manufacturer is set as the initial freshness level for computing item prices. The duration of time that starts at the arrival of a shipment and ends at its assigned expiry date is denoted by E for all the subsets $k = 1, 2, \dots, N(t)$. All the items i in subset k possess identical expiry dates. Therefore an entering customer n at time t seeks to find the subset k' among existing subsets k , $k = 1, 2, \dots, N(t)$. An arriving consumer randomly and uniformly chooses a particular item from the selected subset k' in inventory.

In general, the result of the entire selection procedure in all the modes of operation, at a given demand event point $t = \tau_n$ is described by a discrete-time process $\{S_n(\omega)\}$, where $\{S_n(\omega)\} = \{S_n^{(1)}(\omega), \{S_n^{(2)}(\omega)\}$, which denotes the chosen item $S_n^{(1)}(\omega)$, and the quality of the chosen item $S_n^{(2)}(\omega)$. If $S_n^{(1)}(\omega) \geq 1$, a specific item i in inventory is chosen. The stochastic process jumps coincide with the events of demand $t = \tau_n$, $n = 1, 2, \dots, \infty$.

IV. EXPERIMENTAL RESULTS AND SIMULATION

Extensive computerized runs were carried out in order to estimate the economic effect of using AD in various operational modes: (i) a basic mode where TTI is not available; (ii) a mode in which TTIs are connected to each item and on-line alerts are made whenever the item is damaged, and (iii) a case when TTIs have the ability to predict the real expiry date with some prediction error.

The simulation experiments are made for a specific luxurious fish product with daily average demand of 10 portions. Each portion is sold for 150\$, and expected to be expired in 4 days. The chosen levels of discounts 0%, 5%, 10% and 15% (when freshness is reduced by half) consider a large enough spectrum range at which maximum profits is gained. The model was written in C++.

The following factors were examined on how they affect the expected profits: the TTI operating mode, cost of applying AD c^{AD} , variability of time until an item becomes defective σ_D .

A comparison between the expected and standard deviation of profits with different levels of differentiation intensity has shown according to Table I that a policy of differentiating prices has strong potential to achieve higher profit levels than a policy of a fixed price. At the same time, an excessive differentiation policy may reduce profits. The experiments results show that the expected profit increases by 10% when applying simple TTI's.

TABLE I: PROFIT PERFORMANCE MEASURES FOR DIFFERENT PRICING LEVELS AND USING TTI-BASED ADS.

Fixed Price	Moderate price diff-n	Large price diff-n	Largest price diff-n		Modes of AD
178.12	191.43	180.11	151.74	Avg.	Without
204.68	218.26	220.80	218.58	Sdv.	
177.01	206.37	205.40	182.90	Avg.	Quality status
161.20	157.00	141.21	120.29	Sdv.	
176.11	191.61	176.20	139.72	Avg.	Predicting
160.87	156.56	141.13	122.51	Sdv.	

An unexpected experimental result is that applying complicated TTI's is not cost-effective as it reduces the expected profit by 2.5%. However, according to Fig. 1 the efficiency of TTI technologies in perishable inventory management strongly depends on the costs of TTI. The graph indicates that as long as the technology of detecting spoiled items does not exceed approximately 20% of item cost, it is beneficial to use it.

The experiments also show that applying TTI significantly reduces the variance of profits. It is another benefit of using TTI technologies in the dynamic environment. These results are shown in Table I.

The graph in Fig. 1 indicates that the development of cheaper tags in the future will substantially increase profits in inventory systems with price discrimination.

Fig. 2 graphically shows that the variance of the duration before the expiry date is a vital parameter for reaping

efficiency from TTI technology.

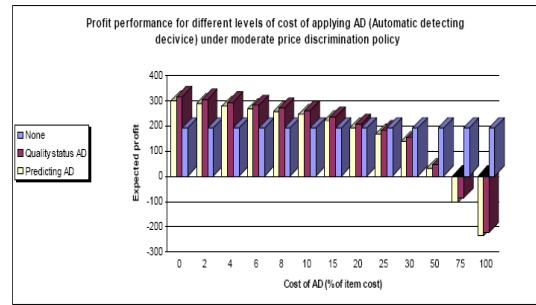


Fig. 1. Profit performance for different levels of TTI costs

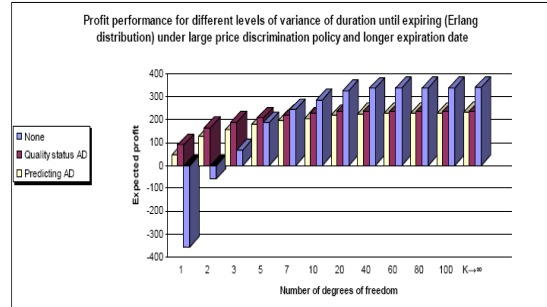


Fig. 2. Profit performance for different levels of duration variance

V. CONCLUSION

The simulation experiments carried out in this study show that in stable deterministic environments the application of TTI is not beneficial. But in dynamic stochastic environments the application of low-cost TTI tags can be highly beneficial. Theoretical analysis of stochastic inventory systems using TTI technology and the influence of other factors on expected profit are attractive directions for future research.

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